

# THE DOUBLET-TRIPLET SPLITTING PROBLEM AND HIGGSES AS PSEUDOGOLDSTONE BOSONS\*

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The doublet-triplet splitting problem is probably the most significant challenge to supersymmetric GUT theories. In this talk, we review potential solutions and their problematic aspects. We also present a complete consistent realization of our preferred solution, higgses as pseudogoldstone bosons, and discuss some distinctive aspects of its phenomenology.

Weak scale supersymmetry might be the solution to the hierarchy problem. Recently in light of the very accurate unification of gauge couplings, supersymmetric GUT theories have received a good deal of attention. However, the status of supersymmetric GUT theories is perhaps not so rosy as it appears. The essential problem is to find a consistent picture of the Higgs sector which requires very light (weak scale) doublets but very heavy (GUT scale) triplets. The purpose of this talk is to emphasize the importance of solving the doublet-triplet splitting problem in establishing the credibility of supersymmetric GUT theories, and to suggest a possible solution.

The outline is as follows. We first review the basic problem and the status of solutions. We will see that most solutions have some problematic feature, and in general are quite complicated. We then present our favored solution, namely Higgses as pseudo-Goldstone bosons. We show that we can build a complete consistent model of a supersymmetric grand unified theory in which the doublet Higgses are light and the triplet Higgses are heavy. We implement a potential which protects the Higgs mass through discrete and gauge symmetries of the potential. The Higgs mass is protected by an accidental global symmetry which is accurate at sufficiently high order in Planck mass suppressed terms to keep the Higgs boson as light as required on phenomenological grounds.

The existence of a complete consistent theory is very important, both from the vantage point of the viability of SUSY GUTs, and as we will discuss, from the view-

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point of phenomenology. We will show that the predictions of our specific model differ significantly from what would be predicted for a “generic” GUT theory, particularly with respect to flavor changing neutral currents.

## 1. Introduction: Doublet-Triplet Splitting, The Quagmire of Supersymmetric GUTs

The major phenomenological motivation for weak scale supersymmetry is the resolution of the hierarchy problem. While the Higgs doublet mass parameters should be in the 100 GeV range, higgsino mediated proton decay argues (in the simplest models) that the Higgs triplet mass parameters are in the  $10^{16}$  GeV range. In the minimal SU(5) model, a parameter must be chosen with accuracy of order  $10^{-13}$ . Although this might be technically natural, it is not very compelling as a solution to the hierarchy problem.

In the absence of simple solutions, we are faced with the question of whether it makes sense to believe a model with a parameter as small as  $10^{-13}$ . Clearly, a better solution is warranted and it is important to explicitly realize it. For those of you who are still not convinced this is a problem, consider the analogous situation for technicolor. Technicolor is *not* ruled out by precision electroweak measurements.<sup>a</sup> It is the difficulty and complication of making a model which can generate masses without excessive flavor changing neutral currents which puts technicolor models in disfavor.

Now consider the status of supersymmetric GUTs. In some ways, the situation is not so different. There are serious fundamental problems from the standpoint of model building. As I have emphasized, the most serious problem is doublet-triplet splitting, but there are other aspects one would like to see simply addressed in a model, such as generating mass, suppressing dangerous flavor changing neutral currents, and suppressing dangerous CP violation. Without a model, it is difficult to believe in the existence of the theory. Without an economical solution to the flavor problem in technicolor theories, they were put in question. But at least the “Higgs” sector works. In supersymmetric GUTs, the theory is stymied at an even earlier stage. What should be emphasized here is that the doublet-triplet splitting problem is not a peripheral issue, but an essential part of the supersymmetric GUT theory. Furthermore, any aspect of the phenomenology which probes the GUT structure can depend in an essential way on this part of the theory. Once we explicitly construct a model, we will exploit it to test the sensitivity of certain predictions, and compare to what might

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<sup>a</sup>For example, the models of Refs. [1,2] contain only a minimal technicolor sector and are therefore acceptable.

have been derived from a “generic” GUT.

## 2. Review of Proposed Solutions

There have been a number of solutions proposed to avoid fine tuning and naturally distinguish the doublet and triplet Higgs masses. They generally have clever group theory structure, which prevents a doublet Higgs mass in the renormalizable Lagrangian. However, as we will see, most models suffer from some unsatisfactory feature. I will review a few of the more promising suggestions here and evaluate their satisfactory and unsatisfactory features.

Before proceeding, let us present some criteria for a good model. First, there should be no unduly small numbers, that is no tuning of parameters. This is a somewhat loosely defined thing—one person’s tuned parameter might be perfectly acceptable to someone else. We suggest the following subjective, but probably adequate, definition. Any number which you would write only in scientific notation is not permitted. The number 0.01 might be OK according to this definition, but  $10^{-5}$  is not.

It should be emphasized that the ratio required between the doublet and triplet Higgs masses is roughly  $(M_G/M_P)^4$ . So if one believes that the allowed Planck suppressed operators are present, it is not sufficient to suppress the Higgs mass in the renormalizable Lagrangian. It must be true that the potential respects whatever symmetry is necessary to fourth order in  $M_P$  suppression.

A third restriction is that the model should permit successful gauge coupling unification within errors, both experimental and theoretical. This is important since the major phenomenological motivation for GUTs is the unification of couplings. This is very constraining—it does not allow for even one additional pair of doublets or triplets.

Now simply stated, GUT theories mandate simple relations between the fields in a single GUT representation. It is very difficult to see how fields from a single representation can have such different masses. Not surprisingly the “solutions” are often very complicated and are not often entirely adequate, though many are quite clever. It is very difficult to find a model in which the doublet-triplet splitting is solved naturally and in which there is no fine tuning.

In Minimal SU(5), the best proposed solution is the Missing Partner Mechanism [3,4]. The idea here is to give the triplet a mass through a Dirac mass term involving a more complicated representation of SU(5) which has the property that it contains a triplet but not a doublet. The 50 is the smallest representation with this feature. One therefore constructs the mass terms

$$W \supset \lambda 5_H \bar{5}_H \langle 75_H \rangle + \lambda' \bar{5}_H 50_H \langle 75_H \rangle$$

so that the triplets, but not the doublets are massive. A model with additional

symmetry to forbid a direct mass term for the  $5$  and  $\bar{5}$  incorporates an additional  $75$  [4].

This is a very nice idea, but seems unlikely to be the resolution of the dilemma. There are several problems with this model. First of all, the large rank of the representations is disturbing. From a theoretical perspective, one has yet to find string theory examples containing these large rank representations. Another problem is that the gauge coupling grows very rapidly, so that the theory is strongly coupled not far above the GUT scale. Although this might be acceptable, it is certainly a problem at the level of nonrenormalizable operators which we discuss shortly.

A further problem is that one cannot leave the states in the remainder of the  $50$  massless, since they contribute like an extra doublet pair to unification, which we know is too much. One can solve this problem for example by adding a mass term  $M50\bar{5}0$  (though this is forbidden by the symmetry of Ref. [4]). But then there is nothing in the symmetry structure of the theory which could forbid the term  $(5)(\bar{5})(75)(\bar{75})/M_p$  which is the product of two allowed terms in the superpotential divided by a third and is therefore allowed by the symmetry, no matter what it is. If one believes Planck suppressed operators consistent with the symmetries are present, the doublet has much too big a mass. This problem is exacerbated in the case the coupling blows up at a low scale, because it is probably the associated strong scale which would suppress such operators.

A more compact implementation of the Missing Partner Mechanism was proposed for Flipped SU(5) [5]. The idea is again to pair up the Higgs with “something else”. Here the something else is a  $10_H$  for the  $5_H$  and a  $\bar{10}_H$  for the  $\bar{5}_H$ , the subscript refers to the Higgs sector to distinguish these fields from the ordinary matter fields. These  $10_H$  and  $\bar{10}_H$  fields are not dangerous because the nonsinglet nontriplet fields are eaten when SU(5) breaks. Hence one has eliminated the necessity for the additional mass term.

The  $10_H$  contains a  $\bar{3}$  but no color singlet weak doublet. The  $10_H$  and  $\bar{10}_H$  get VEVs breaking the  $SU(5) \otimes U(1)$  gauge group to the standard model. The triplet Higgs in  $5_H$  pairs with the triplet in  $10_H$  and the remaining fields in the  $10_H$  are eaten by the massive vector bosons.

This model might work. However, flipped SU(5) is not really a unified model since the gauge group is  $SU(5) \otimes U(1)$  which is not a semisimple group. If it is embedded in a larger gauge group, the problem should be solved in the context of the larger gauge group. The other feature we find disturbing is that there are many assumptions about the vacuum structure. At tree level, there is a D-flat, F-flat direction, and the loop corrections have to be such as to generate the desired minimum. In Ref. [5], the correct ratio of gauge and Yukawa couplings was assumed so that the VEV’s for  $10_H$  and  $\bar{10}_H$  were at the GUT scale while the  $5_H$  and  $\bar{5}_H$  VEV’s are small and the VEVs of an SU(5) singlet generated the “ $\mu$ ” term. It is certainly easier to evaluate the vacuum when it is determined at tree level, as it will be in our preferred model.

Other solutions have been proposed for models which incorporate  $SU(5)$  as a subgroup, for example  $SO(10)$ . The “missing VEV” or Dimopoulos-Wilczek mechanism [6] is probably the most popular  $SO(10)$  solution. The idea is again to pair the triplet and not the doublet Higgs with something else so that the triplet, but not the doublet is massive. In this model, the way this is done is that the VEV aligns so that the triplet, but not the doublet, gets a mass. (This would not have been possible in minimal  $SU(5)$  due to the tracelessness of the adjoint.)

Again, this mechanism seems very nice at first glance, but worrisome at second. For if this were all there was, you would have four light doublets, not two. You need to give the extra doublets a mass, and the problem is how to do this without reintroducing a problem with proton decay. A series of papers by Babu, Barr and Mohapatra [7,8,9,10] showed possible ways to make the DW mechanism into a more complete model.

The first example [7] had two sectors giving VEVs aligning in different orientations, one responsible for the triplet mass, and one responsible for the doublet mass. They thereby achieved strong suppression of proton decay. There was an additional field to complete the breaking of  $SO(10)$  to the standard model, and an additional adjoint to couple the two sectors together (eliminating a massless Goldstone) without misaligning the DW mechanism. The total field content in this model is uncomfortably large –  $3(16) + 3(10) + 3(45) + 2(54) + \bar{16} + 16$ , leading to fairly big threshold corrections and the blowing up of the gauge coupling before  $M_{Pl}$ . Other problems with this particular model was that some operators which would have been allowed by the symmetries of the model needed to be forbidden, and that nonrenormalizable Planck mass suppressed operators could be dangerous.

This last problem was addressed in their second model, where they sacrifice strong suppression of proton decay but generate a natural model, in the sense that they include all operators permitted by their assumed symmetry structure. The field content of this model was  $3(16) + 2(10) + 3(45) + (54) + (1\bar{26}) + (126)$ . Discrete symmetries were sufficient to forbid any unwanted terms from the potential. However, the field content was still quite large, and high rank representations were required.

The third model incorporated a smaller field content and no high rank representations, so it should be more readily obtainable from string models. In this model, the authors achieved the DW form with higher dimension operators, so no  $(54)$  was required. There was a  $\bar{16} + 16$  to complete the breaking to the standard model.

However, without three adjoints, there were intermediate scale pseudo-Goldstone bosons. The authors resolved this problem by cancelling the fairly large corrections to unification of couplings (due to the light charged fields) by large threshold corrections. Although this might work, it is at the edge of parameter space.

Another nice model based on  $SO(10)$  is the model of Babu and Mohapatra [10] which allows for a 10-16 mixing and therefore a Higgs sector which distinguishes the up and down quark masses. However, this model had a few small (but not very

small) parameters, a flat direction and therefore vacuum degeneracy at tree level, extra singlets, and a complicated superpotential.

To summarize, there are some interesting models in the literature, primarily based on clever group theory structure. However most models suffer from one of the following problems.

- There is the problem of actually implementing the potential to get the desired minimum and light Higgses. The minimum can sometimes be destabilized with higher order terms. Also some models have flat directions so the vacuum needs to be carefully thought through.
- It is necessary to ensure the light particle spectrum is compatible with gauge coupling unification. Most solutions rely on pairing up the triplet higgsinos (not doublet) with “something else”. “Something else” can be a problem (with gauge unification).
- The particle representation is cumbersome. This leads to the questions of whether it is derivable from strings or whether the coupling blows up before the Planck scale. In any case, models with large particle content seem unappealing and unlikely.

The problem is clear. Minimal SU(5) relates doublets and triplets! Almost always, the solution relies on a compromise at the edge of parameter space or tuned parameters or setting some couplings to zero in the potential. This is a good introduction to the Higgses as pseudo-Goldstone bosons model which we will argue is an exception to the discussion above. Rather than relying on pairing the Higgs in complicated ways, the theory relies on a spontaneously broken symmetry under which the Higgses are Goldstone bosons. This distinguishes the doublets from the triplets in a very nontrivial way, so that it is natural to obtain light doublets when the remaining fields are heavy. The originally proposed model [11,12,13] involved gauged SU(5) symmetry and a global SU(6) symmetry which was implemented by tuning potential parameters. A better model [14,15,16,17] was later proposed which admits the possibility for justifying the large global symmetry with discrete symmetries. In fact, as we will see, one can construct a simple model to implement this idea [18].

The general idea behind the Higgs as Goldstone scheme is to implement a global symmetry on the Higgs sector which is broken (explicitly) by the Yukawa couplings to matter. The masses of the pseudo-Goldstone bosons are protected from large loop corrections by the nonrenormalization theorem. The Higgs sector can be distinguished by matter parity and consists of the adjoint ( $\Sigma$ ), the fundamental representation ( $H$ ) and the antifundamental ( $\bar{H}$ ), where this refers to their representation under the gauge symmetry. Then additional *global* symmetry is ensured by assuming the superpotential is of the form

$$W(\Sigma, H, \bar{H}) = W(\Sigma) + W(H, \bar{H})$$

At the renormalizable level, this can be achieved by forbidding the coupling  $\bar{H}\Sigma H$ . At higher orders, many other couplings must be forbidden.

By far the nicest implementation of this idea (here we say how we would like the structure to work without explicitly implementing the potential which we do later) is based on extending minimal  $SU(5)$  to the gauge group  $SU(6)$ , with  $SU(6) \otimes SU(6)$  global symmetry. We will refer to this as the  $SU(6)$  model. The Higgses are then an adjoint,  $\Sigma$  which is a  $35 = 24 + 6 + \bar{6} + 1$ , a fundamental  $H$ , in a  $6 = 5 + 1$ , and an  $\bar{H}$ , in a  $\bar{6} = \bar{5} + 1$  where we have given the  $SU(5)$  decompositions. The accidental symmetry is realized if mixing terms of the form  $\bar{H}\Sigma H$  are not present in the superpotential. If the fields  $\Sigma$  and  $H, \bar{H}$  develop VEV's of the form

$$\langle \Sigma \rangle = V \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & -2 \\ & & & & & -2 \end{pmatrix}, \quad (1)$$

$$\langle H \rangle = \langle \bar{H} \rangle = U \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad (2)$$

then one of the global  $SU(6)$  factors breaks to  $SU(4) \otimes SU(2) \otimes U(1)$ , while the other to  $SU(5)$ . Together, the VEV's break the gauge group to  $SU(3) \otimes SU(2) \otimes U(1)$ .

The Goldstone bosons (GB's) coming from the breaking  $SU(6) \rightarrow SU(4) \otimes SU(2) \otimes U(1)$  are (according to their  $SU(3) \otimes SU(2) \otimes U(1)$  transformation properties):

$$(\bar{3}, 2)_{\frac{5}{6}} + (3, 2)_{-\frac{5}{6}} + (1, 2)_{\frac{1}{2}} + (1, 2)_{-\frac{1}{2}}, \quad (3)$$

while from the breaking  $SU(6) \rightarrow SU(5)$  the GB's are

$$(3, 1)_{-\frac{1}{3}} + (\bar{3}, 1)_{\frac{1}{3}} + (1, 2)_{\frac{1}{2}} + (1, 2)_{-\frac{1}{2}} + (1, 1)_0. \quad (4)$$

But the following GB's are eaten by the heavy vector bosons due to the supersymmetric Higgs mechanism (the gauge symmetry is broken from  $SU(6)$  to  $SU(3) \otimes SU(2) \otimes U(1)$ ):

$$(3, 1)_{-\frac{1}{3}} + (\bar{3}, 1)_{\frac{1}{3}} + (3, 2)_{-\frac{5}{6}} + (\bar{3}, 2)_{\frac{5}{6}} + (1, 2)_{\frac{1}{2}} + (1, 2)_{-\frac{1}{2}} + (1, 1)_0. \quad (5)$$

Thus exactly one pair of doublets remains uneaten which can be identified with the Higgs fields of the MSSM. One can show that the uneaten doublets are in the following combinations of the fields  $\Sigma, H, \bar{H}$ :

$$h_1 = \frac{Uh_\Sigma - 3Vh_H}{\sqrt{9V^2 + U^2}}, \quad (6)$$

$$h_2 = \frac{U\bar{h}_\Sigma - 3V\bar{h}_{\bar{H}}}{\sqrt{9V^2 + U^2}}, \quad (7)$$

where  $h_H$  and  $\bar{h}_{\bar{H}}$  denote the two doublets living in the  $SU(6)$  field  $H$  and  $\bar{H}$ , while  $h_\Sigma$  and  $\bar{h}_\Sigma$  denote the two doublets living in the  $SU(6)$  adjoint  $\Sigma$ . In order to maintain the correct prediction for  $\sin^2 \theta$ , we need to have  $\langle \Sigma \rangle \sim M_{GUT}$ ,  $\langle H \rangle = \langle \bar{H} \rangle > \langle \Sigma \rangle$ .

It is important to note that the triplets in  $H$  and  $\bar{H}$  are eaten and not dangerous. Only the  $\Sigma$  triplet needs to be made heavy in the superpotential to avoid proton decay.

Having established the desired vacuum and symmetry structure, we now have to face the question of whether such a model exists. In fact, we would once again like to see a model with the more stringent requirement that the Higgs is massless up to order  $(M_G/M_{Pl})^4$ . Furthermore we will take the point of view that only gauge and discrete symmetries are exact. To obtain the desired structure therefore requires an accidental symmetry which is respected in nonrenormalizable terms to fourth order in  $1/M_{Pl}$ . We also require that the potential has no flat directions so that the minimum is determined and is the desired one. In order to show that it is a viable model we also require that the model can be extended to fermion masses. We emphasize that the existence of such a model is important because there are no other complete consistent and simple models. The model [18] we constructed has simple field content, no small parameters, and the above properties.

Before actually presenting an example of a model, let us first understand the difficulty. Suppose the potential is such that the minima of  $\Sigma$  and  $H$  are determined. We then require at least four terms in the potential (one can not have nonzero VEVs for  $\Sigma$  and  $H, \bar{H}$  with just three terms).

$$\begin{aligned} & \frac{1}{M_{Pl}^{a-3}} \text{Tr} \Sigma^a + \frac{1}{M_{Pl}^{b-3}} \text{Tr} \Sigma^b, \\ & \frac{1}{M_{Pl}^{2c-3}} (\bar{H}H)^c + \frac{1}{M_{Pl}^{2d-3}} (\bar{H}H)^d \end{aligned}$$

But then the symmetries allow

$$\frac{1}{M_{Pl}^{2c+b-a-3}} (\bar{H} \Sigma^{b-a} H) (\bar{H}H)^{c-1}$$

Notice that the dangerous term involving both  $\Sigma$  and  $H$  is suppressed by precisely the ratio of two superpotential terms. But at the minimum, all the terms in the potential should be of the same order of magnitude. This means that the ratio of fields is of the same order of magnitude as the ratio of the coefficients of the operators. To get

the required suppression of the Higgs mass would then require a ratio of coefficients of order  $(M_W/M_{Pl})!$  This is precisely what we are trying to avoid—that is tuning the Higgs mass to be small.

So we need to explore the possible loopholes in the above reasoning. One possibility is to add more fields, so that one cannot make a holomorphic function with positive powers of the fields which is allowed by the existing symmetries. But to eliminate the flat directions then requires more terms in the superpotential. Furthermore we do not want very high dimension operators in the  $\Sigma$  potential (because when the VEV is of order  $M_G$ , the triplets in the  $\Sigma$  will get too small a mass, suppressed by  $(M_G/M_{Pl})$  to a large power). Also, the dimensions of the operators in the  $\Sigma$  or  $H$ ,  $\bar{H}$  potential should not be very different, or else the ratio of couplings will be large (or small) so that all terms in the potential can be of the same order of magnitude at the minimum. But with sufficiently many fields and low dimension terms in the potential to eliminate flat directions, one can generally make invariant operators by more complicated extensions of the above argument. We found no examples where we obtained a satisfactory minimum at tree level and all dangerous operators were forbidden.

We conclude we need a small number to make a model. Fortunately we know there exists a small number, namely the ratio of the weak scale to the Planck scale. Even without knowing how this ratio arises, we know this small number is present in any satisfactory model.

The alternative for a small number is 0. One can also construct models which incorporate fields with zero expectation value, so that the dangerous terms involving the  $\Sigma$  and  $H$  fields vanish.

Several models involving one or the other of these ideas were presented in Ref. [18]. We present the simplest of the examples in the following section.

### 3. A Model

The field content is the minimal field content required, namely  $H$ ,  $\bar{H}$ , and  $\Sigma$ . We impose an additional discrete symmetry  $\bar{H}H$ :  $\bar{H}H \rightarrow e^{2\pi i/n} \bar{H}H$ .

The superpotential is

$$W = \frac{1}{2} M \text{Tr} \Sigma^2 + \frac{1}{3} \lambda \text{Tr} \Sigma^3 + \alpha \frac{(\bar{H}H)^n}{M_{Pl}^{2n-3}}$$

and the potential after the inclusion of the soft breaking terms is given by

$$\begin{aligned} V(\Sigma, \bar{H}, H) = & \text{Tr} |M\Sigma + \lambda\Sigma^2 - \frac{1}{6}\lambda \text{Tr} \Sigma^2|^2 \\ & + \frac{n^2\alpha^2}{M_{Pl}^{4n-6}} (\bar{H}H)^{2n-2} (|H|^2 + |\bar{H}|^2) + \end{aligned}$$

$$\begin{aligned}
& Am\lambda \text{Tr}\Sigma^3 + A'm\alpha \frac{(\bar{H}H)^n}{M_{Pl}^{2n-3}} + BMm\Sigma^2 \\
& + m^2(\text{Tr}\Sigma^2 + |H|^2 + |\bar{H}|^2) + \text{D-terms}
\end{aligned} \tag{8}$$

Notice that at the minimum for  $H$ , a high dimension operator is balanced against the soft supersymmetry breaking terms. There is a minimum with

$$\langle H \rangle = \langle \bar{H} \rangle = \left( \frac{m}{M_{Pl}} \right)^{\frac{1}{2n-2}} M_{Pl}$$

For  $n = 4, 5, 6$ , we get  $\langle H \rangle \approx 1.5 \cdot 10^{16}, 6 \cdot 10^{16}, 2 \cdot 10^{17}$  GeV. The first mixing term allowed by  $Z_n$  is  $\frac{1}{M_{Pl}^{2n-2}}(\bar{H}H)^{n-1}(\bar{H}\Sigma H)$ . This gives mass to the pseudo-Goldstone bosons which is less than the weak scale, and therefore safe. Notice it was essential that  $H$  mass term was small for naturalness of the model. This was only permitted because the triplets in  $H$  and  $\bar{H}$  were eaten.

We emphasize that this model had no fields other than an adjoint, fundamental, and antifundamental, which we expect to be present in any  $SU(n)$  model which breaks to the standard model and gives the necessary fermion masses. However, to accommodate fermion masses in our context and to generate a successful mass texture, we found it useful to incorporate an additional adjoint field. With an additional discrete  $Z_3$  symmetry (under which  $\Sigma_1 \rightarrow e^{2\pi i/3}\Sigma_1$ ,  $\Sigma_2 \rightarrow e^{-2\pi i/3}\Sigma_2$ ,  $\bar{H}H \rightarrow \bar{H}H$ ), the superpotential takes the form

$$M\text{Tr}\Sigma_1\Sigma_2 + \frac{1}{3}\lambda_1\text{Tr}\Sigma_1^3 + \frac{1}{3}\lambda_2\text{Tr}\Sigma_2^3 + \frac{\alpha}{M_{Pl}^{2n-3}}(\bar{H}H)^n$$

This is useful for constructing a mass model in which the masses and mixing angles are naturally of order of magnitude of the ratio of VEVs and the Planck scale [18]. These are enforced through discrete symmetries acting on the fermions. One interesting aspect of these models is that the top quark is in a distinct representation from the other up type quarks [16,18]. This is required in order to give a renormalizable Yukawa coupling to the top quark so it can get a sufficiently large mass. Another interesting aspect of the mass models is that  $b - \tau$  unification occurs naturally, because there is a unique operator which gives a mass to the  $b$  and  $\tau$  (so there are no Clebsches distinguishing the mass).

#### 4. Phenomenology

Because of the enlarged gauge group and the restricted Higgs sector, the phenomenology of the  $SU(6)$  model operates very differently from the minimal  $SU(5)$  model. However, since there is probably not a minimal  $SU(5)$  model, since any GUT model which solves the doublet-triplet splitting problem is likely to have additional

structure, it is important to explore the phenomenology of a model with a satisfactory Higgs sector. These predictions can differ significantly from “generic” results.

Here we focus on the violations of lepton flavor in supersymmetric unified theories [20]. Barbieri, Hall, and Strumia point out that in supersymmetric GUTs there is large flavor violation in the *lepton* sector due to  $\lambda_t$  and that it is communicated to physics at low energies through the scalar partners. In principle, one can test SUSY GUTs through flavor changing processes, such as  $\mu \rightarrow e\gamma$ . They worked out the predictions for the minimal GUT theories. However, the SU(6) model predictions look very different as a function of the couplings and SUSY soft parameters from the minimal SU(5) case. Let us focus for the moment on what distinguishes the predictions of the SU(6) model from those of a “generic” GUT. First of all, the top quark is in a different gauge representation from the other up type quarks, so in principle there are additional flavor changing effects from *gauge* interactions. It turns out however these are smaller than those due to the top Yukawa coupling. Second, there are different possible contractions of higher dimension operators so there are different mixing angles for the leptons and down type quarks, reducing the predictability of the flavor changing lepton process (related to unknown mixing angles). Another distinguishing aspect of our model is that there are additional potential flavor dependent Yukawa couplings from interactions with heavy fermions (which are necessarily present in the theory for the reason of anomaly cancellation). These can however be naturally suppressed by discrete symmetries [19].

What turns out to be the most important numerical difference to the prediction is the larger gauge group requiring larger representations. This makes everything run much faster, since essentially the counting factors on loop diagrams are bigger.

Let us now consider the predictions. First we look at the top Yukawa coupling,  $\lambda_t$ . There is an upper bound in this model from two things—first we require the top Yukawa to be perturbative up to the Planck scale, which gives a bound  $\lambda_t < 1.12$  (compared to  $\lambda_t < 1.56$  in minimal SU(5)). Second, we require that the stau mass remains positive. In minimal SU(5) the running is slower, so there is usually no such a constraint. There is also a lower bound on  $\lambda_t$ . This comes from requiring that  $b$ - $\tau$  unification works. In our model, it is easy to make  $b$ - $\tau$  unification work to a level of a few percent, but difficult to make it work with greater than 10% error without completely destroying the prediction. With only this much leeway,  $\lambda_t$  cannot be too small, greater than about 0.9.

Now with  $\lambda_t$  constrained to be so small, minimal SU(5) would predict unobservably small flavor violations. However, because in SU(6) the masses run so much more quickly, we find we still predict potentially testable flavor changing lepton processes over most of parameter space. The details of this analysis will be given in Ref. [19]. We find for comparable  $\lambda_t$ , the rate for  $\mu \rightarrow e\gamma$  is an order of magnitude greater than

for minimal  $SU(5)$ .

## 5. Conclusions

Supersymmetric GUTs seem very nice, but they are theoretically problematic. The most important issue from a model building perspective is to understand how the doublet-triplet splitting problem can be resolved. Without a solution, the existence of the theory is difficult to support.

Most solutions have some problem. However, the Higgs as pseudo-Goldstone boson solution appears to be an exception. We have constructed a rather simple model where the Higgs is light because of additional accidental global symmetry present in the theory. This gives a natural resolution to the doublet-triplet splitting (and the  $\mu$ ) problem. We have a complete consistent example. Moreover, with an explicit realization of the model, one can explore its phenomenological consequences. This gives a new and maybe even more realistic perspective on low energy phenomenology. It is important to understand the range of possibilities which follow from a complete model.

One might also interpret the difficulty in constructing GUT models as indicative that the theory is not unified below the Planck scale. In this case, we will ultimately want to understand the discrepancy between the unification scale and the Planck scale. If we are to apply the same standards applied to the GUT models, we would also want to better understand the string vacuum and resolve the problem of moduli proliferation and the associated vacuum degeneracy. After all, all the solutions appeared beautiful until one tried to implement them explicitly in realistic nonfinetuned natural models which gave the correct value for  $\sin^2 \theta$ .

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